Anisotropic Virtual Coupling with Energy-Based Deflection for Palpating Inhomogeneous Compliant Objects

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ABSTRACT
Virtual coupling, a spring-damper system between the haptic probe and its virtual representation, the proxy, is one of the most common approaches for haptic rendering. We have extended the virtual coupling by updating the spring stiffness, sometimes used to simulate compliance of a material, depending on the direction between the proxy and the probe. This anisotropic variation of the stiffness is used in exploring inhomogeneities beneath the surface allowing detection of rigid structures even when they are obscured by another structure beneath the surface. In addition, we also compensate for the energy variation of the spring to maintain passivity and increase realism. User studies were performed to survey the success rate in the detection of obscured rigid bodies beneath the surface with the modified virtual coupling algorithm and the improvement of shape perception for sub-surface objects with the additional energy compensation term providing gradient information. We also discuss potential benefits of the proposed methods as basic extensions to well-known haptic rendering algorithms which are both simpler and yield improved performance over traditional deformation simulation techniques.

1 INTRODUCTION
Integrating haptics into virtual applications to achieve higher immersion has become popular in recent years. In addition to developments in hardware solutions and research in psychophysics examining the perception of touch, different algorithms have been developed to evaluate the feedback force under the name of haptic rendering. Virtual coupling [5], a spring-damper system between the haptic probe and its virtual representation, the proxy, has been one of the most popular approaches since it simplifies the problem of ensuring stability of the haptic display [9].

Different versions of virtual coupling [17, 19] propose simulating compliant material properties by updating the parameters of the spring-damper system depending on the position of the haptic probe or proxy. Although this approach has been proven to work for simulating the hardness of an object, it is not sufficient for providing comprehensive information about the underlying structures. The most common use, for example, is to simulate the hardness of a surface model by retrieving material properties on the surface. While proxy-based volume haptics [10] can provide additional information about the structures underneath the surface, the principle of using one position, proxy or probe, is not sufficient for exploring inhomogeneous data with varying material properties.

To address this problem, we propose combining anisotropy, the property of being direction dependent, with virtual coupling, we call this Anisotropic Virtual Coupling (AVC). A ray cast in the direction from the proxy to the probe is used to modify the stiffness of the spring-damper system. In the case of surface exploration, this technique uses both the stiffness of the surface and the underlying information along the ray direction. This allows, in addition to feeling surface properties, feeling rigid bodies, for instance, underneath the surface even when they are obscured by another rigid body.

Updating the virtual coupling spring stiffness by the proxy-probe direction results in energy changes in the system. In order to maintain energy conservation for stability purposes, we compensate for the energy changes through additional forces fed back to the haptic device and the proxy update algorithm. We call this latter option Anisotropic Virtual Coupling with Energy Conservation (AVCEC).

We have performed user studies to examine the performance of these two techniques. Our contributions can be summarized as:

• an algorithm providing information, in addition to the surface properties, about structures beneath the surface allowing the user to feel structures even when they are obscured.
• integration of energy conserving components to maintain passivity, increase realism and provide better shape perception.
• a user study; an experiment for AVC evaluating whether a rigid box obscured by a rigid flat plate can be located while exploring a surface covering both the rigid box and the rigid plate.
• another user study; an experiment for AVCEC studying whether the additional feedback from energy conservation, provided as gradient information, improves shape perception for structures beneath the surface.

2 RELATED WORK
After the idea of virtual coupling was introduced in [5], it was employed in one of the most common haptic rendering algorithms the god-object algorithm [19] which was extended to a more general approach [17] with additional surface properties such as texture and friction. The basic principle was further exploited to achieve various advances by researchers in the field. In [13] virtual coupling was adapted to 6-dof rendering with texture and friction while hand grasping was simulated in [8]. The principle was not only used with surface models but integrated to volumetric data as well [2, 10]. Basic computer graphics algorithms like shading and bump-mapping were adapted for haptic rendering to simulate surface texture and friction in [7]. Anisotropy and inhomogeneity was considered for friction forces in [3].

The essence of the algorithm we propose depends on updating the virtual coupling spring stiffness depending on the data beneath the surface. This dynamic stiffness change requires consideration of energy change to maintain energy conservation. In [1, 18] energy conservation for a changing scalar field in the case of haptic rendering was discussed and different texture rendering algorithms were analysed with respect to energy conservation in [1]. The compensation for the energy change for a varying stiffness field results in additional forces which can be rendered as, for example, lateral forces on the surface [14].
Lateral forces on a surface have been surveyed in different contexts such as haptic texture rendering [11] by perturbing the direction of the reaction force or shape perception [6, 12, 15, 16]. In [16], it was shown that the force cues dominated, regardless of surface geometry, in the identification of such shape features as bumps or holes. This result was further researched in [6] which found a higher contribution from position cues for more convex high arches by user studies with decoupled force and position cues. [12] also surveyed effects of force shading on perception of bumps and holes suggesting that the effects of force shading on perception were dependent on curvature and the number of polygons.

Another related issue which needs to be addressed is the sensing of haptic edges depending on the force constancy theorem [4]. Surfaces with varying stiffness cause unintentional changes in the hand movement, conflicting with the topology of the surface. A modification for the traditional haptic rendering approach was applied to eliminate this conflict with the topology of the surface.

Our use of virtual coupling is different from the scenarios described above and our use of the energy compensation component affects both the proxy and the probe, as explained later. The shape perception studies we performed were based upon geometrical shapes, such as circle, triangle and other polygons, focusing on the detection of edges rather than structures on a surface such as bumps and holes as in the case of studies above.

3 Anisotropic Virtual Coupling — AVC

The basic principle of virtual coupling is to employ a spring-damper system between the haptic probe and the proxy. This provides better control of the force feedback by restricting the proxy movement with respect to constraints in the virtual environment and evaluating the force via the spring-damper system.

The most popular approach for simulating material properties has been to determine the stiffness and damping parameters of the virtual coupling depending on the position of the probe or proxy. This allows the user to feel the object at the contact region only. The most common use has been in exploring surface models [17, 19] although the method has also been applied to volumetric data [10] as well. With this traditional approach it is possible to feel the hardness of a surface or an underlying rigid body as well. Exploring a more complex scenario, such as having a rigid body obscured by another one, figure 1, requires a more complete approach.

We propose modulation of the stiffness of the material by casting a ray from the proxy to the probe. The ray traverses the data until it hits a rigid body, the ground or exceeds a predetermined maximum length. The length of the ray is used to update the stiffness depending on the relation between material and geometric properties of a spring:

\[ k = \frac{EA}{l} \]  

where \( E \) is the modulus of elasticity, \( A \) is the cross-sectional area of contact and \( l \) is the length of the spring. In the case of exploring a surface, we relate the stiffness of the surface to an initial length. The ratio between the length of the ray cast in the proxy-probe direction and the initial length is then used for modulating the surface stiffness to update the virtual coupling stiffness. The closer the rigid body is, the shorter the length of the ray, and so the stiffer the spring becomes. This modulation introduces additional information about structures beneath the surface while maintaining the surface properties. This allows the user to feel the surface and rigid bodies along the movement direction simultaneously. Rigid bodies obscured by another rigid body along the movement direction can thus be felt by approaching from a different direction. Because of its dependence on direction, we call this algorithm Anisotropic Virtual Coupling, AVC.

One requirement for the current implementation of the algorithm to detect the obscured rigid bodies is that, for at least at one point on the surface, there should not be any obstacle between the proxy and the rigid body along the proxy-probe direction. In order to guarantee such a point for a partially obscured rigid body as in figure 1, a varying proxy-probe direction with the direction of movement is required during surface exploration. This variation of the proxy-probe direction is achieved by surface friction in the proxy update algorithm [17]. Without friction the proxy becomes a projection of the probe on the surface resulting in a proxy-probe ray direction which is always parallel to the surface normal, which prevents the detection of the partially obscured rigid body in figure 1. In the case of a rigid body being completely obscured, for instance completely covered by another rigid body, extensions exploiting ray casting techniques as in volume rendering need be applied to make the rigid body detectable, as described in section 4.4.

4 AVCoupling with Energy Conservation — AVCEC

The essence of the virtual coupling depends on the well known Hooke’s Law, which defines the spring force to compensate for potential energy change stored in the spring. During compression of a spring the potential energy change is given by the stiffness multiplied by the compression distance, resulting in a force balancing the change. In the case of a varying spring stiffness, if a physics-based approach is to be followed, the potential energy change needs to be taken into account. Energy changes not being compensated for result in energy sources and sinks during exploration of a surface and affect both the stability of the system and the realism of the forces. This issue, however, has mostly been ignored in the deployment of virtual coupling in exploration of surfaces with varying stiffness.

The proposed algorithm in the previous section, AVC, updates the virtual coupling spring stiffness dynamically depending on the position of the proxy and probe. We consider the potential energy changes during the evaluation of the force and call this technique Anisotropic Virtual Coupling with Energy Conservation, AVCEC.

4.1 Energy Variation

Potential energy stored in a spring is given by the area under the force curve in a force-displacement graph and in our case can be formulated as:

\[ E(\vec{x}_{py}, \vec{x}_{pb}) = \int_{0}^{D} k(\vec{x}_{py}, \vec{x}_{pb}) dr = k(\vec{x}_{py}, \vec{x}_{pb}) \frac{D^2(\vec{x}_{py}, \vec{x}_{pb})}{2} \]  

where \( E \) is the energy function depending on the proxy position on the surface, \( \vec{x}_{py} \) in \( \mathbb{R}^2 \), and the direction from the proxy to the
The energy change can be evaluated as:

\[ \nabla E(\vec{x}_p, \vec{x}_b) = \nabla k(\vec{x}_p, \vec{x}_b) \frac{D^2(\vec{x}_p, \vec{x}_b)}{2} + k(\vec{x}_p, \vec{x}_b) D(\vec{x}_p, \vec{x}_b) \nabla D(\vec{x}_p, \vec{x}_b) \]

\[ = \nabla k(\vec{x}_p, \vec{x}_b) \frac{D^2(\vec{x}_p, \vec{x}_b)}{2} + k(\vec{x}_p, \vec{x}_b) D(\vec{x}_p, \vec{x}_b) \vec{d} \]

where \( \vec{d} \) is the unit vector along the proxy-probe direction. If one wants to compensate for the energy change by negating equation 3:

\[ -\nabla E(\vec{x}_p, \vec{x}_b) = -k(\vec{x}_p, \vec{x}_b) D(\vec{x}_p, \vec{x}_b) \vec{d} \]

\[ -\nabla k(\vec{x}_p, \vec{x}_b) \frac{D^2(\vec{x}_p, \vec{x}_b)}{2} \]

where the first term refers to the well known spring force depending on Hooke’s Law and the second term refers to the force for energy compensation. \( \nabla E \) becomes a vector in \( \mathbb{R}^3 \), affecting both the probe and the proxy movements. We therefore simplify it by considering the probe and the proxy separately, the proxy movements being modulated by this energy compensation force and the probe being affected through force feedback.

4.2 Proxy Modulation

The energy compensation component related to the proxy movement, \( \vec{F}_{py} \), results in a lateral force at the surface since the proxy movement is limited to the surface [14]. \( \vec{F}_{py} \) and can be calculated by:

\[ \vec{F}_{py} = -\nabla \vec{x}_p k(\vec{x}_p, \vec{x}_b) D(\vec{x}_p, \vec{x}_b) \frac{D^2(\vec{x}_p, \vec{x}_b)}{2} \]

The partial derivative with respect to proxy movement is evaluated using a 2D Sobel kernel, figure 2. The boundaries of the kernel during exploration of the surface are shown by green lines in the figure. The application of the kernel to the proxy which is restricted to the surface during contact results in a lateral force, \( \vec{F}_{py} \) shown as the black arrow in the figure, which is then used in the algorithm to calculate the proxy position [17].

4.3 Probe Deflection

Unlike \( \vec{F}_{py} \), the second component, \( \vec{F}_{pb} \), is a three dimensional force since it refers to the partial derivative of the energy with respect to haptic probe movement which can be formulated as:

\[ \vec{F}_{pb} = -\nabla \vec{x}_p k(\vec{x}_p, \vec{x}_b) D(\vec{x}_p, \vec{x}_b) \frac{D^2(\vec{x}_p, \vec{x}_b)}{2} \]

The partial derivative is calculated using a 2D Sobel kernel mapped to a sphere around the proxy point. The blue lines in figure 2 represent the boundaries of the kernel used to calculate the \( \vec{F}_{pb} \), which is shown as a yellow arrow. The resulting torque is converted to a force and then added to the coupling feedback.

4.4 Possible Extensions and Limitations

The described technique modulates the stiffness of the surface by the length of the ray cast, the distance to the closest rigid body. The main idea, however, can be extended to sampling volume information along the ray to use during modulation. Taking the average or the maximum stiffness along the ray, analogous to ray casting in volume rendering, are among the several possibilities which can be employed to detect rigid bodies completely obscured from all directions by another obstacle.

The range for the length of the ray cast is a parameter to be determined affecting the stiffness modulation as well as which rigid bodies to consider underneath the soft tissue. Prior knowledge about the data can be used for determination of this range. Rigid bodies entering and leaving this range and large distances between the ground and rigid bodies may result in discontinuities in stiffness as well. These discontinuities are smoothed during evaluation of the energy compensation term since the kernels used to calculate the stiffness gradient have a smoothing effect. By varying the size of these kernels the amount of smoothing can be adjusted. Where more smoothing is needed, a low pass filter can be employed around the neighbourhood of the proxy by sending more rays parallel to the same direction used to calculate the stiffness. The use of a low pass filter is illustrated by the black lines in figure 1.

5 Evaluation

We performed a user study with two different experiments: First an AVC Experiment to determine whether the position of a rigid box, obscured by a rigid flat plate beneath a surface, can be detected using our algorithm AVC. Second an AVCEC Experiment to determine whether the shape perception of rigid bodies beneath a surface can be improved by AVCEC.

5.1 Method

The experiments were performed in a haptic workstation equipped with a Desktop Phantom as illustrated in figure 3. In both experi-
ments, for each stimulus the subjects were presented with an opaque virtual box with hidden structures inside. The left side of the figure 4 shows the visual representation presented to the subjects in the AVC experiment, while the same box without markings on its top surface was rendered during AVCEC experiment. The inner content of the box was determined by the experiment type, as explained below. The orientation of the box was adjusted such that the palpation occurs on the axis perpendicular to the desk. To prevent visual cues relating to the strain applied, the appearance of the box did not change in response to the compression and the haptic probe was rendered as a sphere which remained on the surface of the box during contact in all situations. The subject was also prevented from seeing the real hand position under the semi-transparent mirror by installing a sheet of white paper under the mirror and setting the background colour to bright white.

The top surface was chosen to be a square with an edge length of 15 cm and the height of the box was set to 7 cm throughout the experiments. It was observed during pilot studies that continuous use of the haptic device with high stiffness values can result in overheating of the motors. Based on this, 70 N/m was determined as a suitable reference stiffness for the top surface of the virtual box.

Ten subjects took part in the experiments, 9 male and 1 female. They were undergraduate students and researchers aged between 27 and 41 years (mean age was 30). 7 of the subjects had tried a haptic device on a few occasions previous to the experiments and 3 had used them quite often. All subjects had normal or corrected to normal vision, one subject was left handed. They received no compensation for taking part in the experiments.

Before the experiments began background information was obtained from each subject. They then reviewed written instruction material and were instructed about the equipment and the tasks to be performed. Before each experiment they also completed a set of practice trials for the corresponding experiment and at the end of the whole experiment they completed a questionnaire providing qualitative information about their experience. Total participation time lasted 1 hour, on average, including the introductory part.

5.2 AVC Experiment

The task for the subjects in this experiment was to find where a rigid box obscured by another rigid flat surface was hidden. Both rigid box and the rigid flat surface lie under an opaque surface of the virtual box shown in the right side of the figure 4. A two dimensional square plate was chosen as the obscuring rigid flat surface and placed in the middle of the virtual box and parallel to the top surface. Under the square plate a rigid box with a square top surface of half the size of the square plate was placed. This allowed, from the top view, the rigid box to lie only in one quarter of the square plate. In the right side of the figure 4, the rigid box lies under the bottom-left quarter of the square plate. While exploring the top surface of the virtual box by traditional virtual coupling approaches, it is impossible to feel the rigid box under the square plate. With AVC, however, we expected the subjects to be able to feel the rigid box while approaching towards the square plate on the surface from different directions since the ray along the proxy-probe direction can theoretically intersect with the rigid box before it intersects with the square plate.

To achieve a non-vertical proxy-probe direction, friction is needed on the explored surface, otherwise AVC behaves like a traditional virtual coupling algorithm. The static friction coefficient was set to 0.4, and dynamic friction to 0.7 and the algorithm in [17] was used to evaluate the proxy position. The edge of the square plate and the top surface of the rigid box were chosen to be 6 and 3 cm, respectively. The height of the rigid box was 0.5 cm. The square plate was located 2 cm below the top surface of the virtual box and the rigid box 2 cm below the plate. Since the depth difference between the ground and the rigid structures results in disturbing discontinuities, a $5 \times 5$ Gaussian filter with a grid size of $1.875 \text{ mm} \times 1.875 \text{ mm}$ was employed as explained in section 4.4.

The subjects were presented with 40 trials in total, and in each trial the rigid box was randomly placed under one of the four quarters of the square plate. The four quarters of the top surface of the virtual box were also divided into four quadrants, as in the left side of the figure 4, and the subjects were asked to make a lateral movement from the edge towards the centre in all four quarters. The task was to identify which region of the square plate the rigid box was hidden beneath, and they made their choice by pressing a numerical pad, displayed to one side of the screen, using the haptic device.

5.3 AVCEC Experiment

The principle of energy conservation depending on a physics-based technique promises more realistic feedback. The additional force component for energy compensation provides stiffness gradient information. During our trials with AVCEC, we observed that it facilitates detection of boundaries and edge-following, probably because of the additional gradient information. We, therefore, designed an experiment to survey the effect of the energy compensation component on shape recognition. Instead of placing two rigid bodies in the opaque virtual box, as in the previous experiment, we placed only one rigid flat surface, a 2 dimensional rigid plate of one of a number of shapes, and asked the subjects to identify the shape of the plate. The rigid plate was always kept parallel to the top surface of the virtual box while it varied among five shapes: circle, triangle, rectangle, pentagon, and hexagon. The visual representation to subjects was similar to the previous experiment, left side of the figure 4, though without the surface markings.

There was no static friction on the top surface of the virtual box explored surface while the dynamic friction was set to 0.7. The rigid plate was placed 3 cm below the top surface. During informal pilot studies, it was observed that having the same edge length for different shapes results in different surface areas providing additional size cues to the shape. In order to eliminate these cues, the same area was used instead of the same edge length among the shapes. A random orientation, always keeping the plate parallel to the top surface, was applied for each trial to prevent subjects using edge orientation cues and memorizing the positions of the corners. A Gaussian filter with the same parameters as in the previous experiment was deployed as explained above. For energy conservation, a $9 \times 9$ Sobel kernel was employed for both $F_{py}$ and $F_{pb}$. The grid size for the Sobel kernel of $F_{py}$ was chosen as $3.75 \text{ mm} \times 3.75 \text{ mm}$ and the step angle between the rays used in the Sobel kernel of $F_{pb}$ was chosen as $5^\circ$.

The evaluation was performed as a within-subjects design with two conditions; with and without energy conservation, which will be referred to as energy condition and non-energy condition respectively. The experiment was performed over two separate sessions.
where each condition was carried out once. The presentation order of the conditions for each subject was balanced and a trial was performed before each condition. The frequency of the five different shapes was balanced but randomly distributed to 25 trials for each condition. The subjects were told to guess the shape in the virtual box among the five shapes by pressing a pad displayed to one side of the screen using the haptic device. The subjects were instructed that the main task was to correctly determine the shape while the time for each trial was also important.

6 Results

In case of experiment AVC, analyzing the percentage of correct answers for the subjects showed a mean value of 98% success rate with a standard deviation of 3.8%. 7 subjects out of 10 found the correct region among the four in all 40 trials.

For the experiment AVCEC, the percentage success rate and time spent during the trials were compared both for each shape type individually and in total. One can observe higher success rates for the energy condition for all shape types in figure 5(a). In order to perform a statistical analysis, non-parametric tests were used since the data deviated from a normal distribution. A Wilcoxon signed-rank test showed that the energy condition has significantly higher success rate than the non-energy condition for the shapes circle \( (Mdn_{\text{energy}} = 100\% , Mdn_{\text{non-energy}} = 80\% , z=-2.84, p<0.01, r=-0.39) \) and rectangle \( (Mdn_{\text{energy}} = 90\% , Mdn_{\text{non-energy}} = 20\% , z=-2.69, p<0.01, r=-0.38) \). The analysis for the whole data set resulted in significantly higher performance for the energy condition \( (Mdn_{\text{energy}} = 66\% , Mdn_{\text{non-energy}} = 28\% , z=-2.81, p<0.01, r=-0.18) \) also. There was no significant difference observed for the pentagon and hexagon. The same type of analysis was performed for the time spent for each condition and, as in the success rate, a significant improvement was observed for circle, triangle and rectangle. Although higher mean values of time spent were observed for all shape types in the non-energy condition, figure 5(b), not having a statistical significance for all types can be explained by checking each subject’s results individually and qualitative information obtained from the post-test questionnaire.

The subjects were asked whether they developed a strategy to recognize the shapes of the objects in experiment AVCEC. The comments mostly described following the edges in the energy condition while for the non-energy condition they had to make either a zigzag movement trying to find the edges or sweep all over the surface systematically as in a rasterization process. The strategies for the latter condition took more time for the subjects resulting in the higher mean value in time measured. The lack of a significant difference can be explained by informal observation of some of the subjects giving up due to the difficulty and simply guessing the shape which ended up in shorter times in the non-energy condition.

The subjects were also asked to rank the difficulty of the experiments within the range 1 and 5; 1 being very difficult and 5 very easy. The mean rank for the experiment AVC was 4, being easy, while in the case of the experiment AVCEC, it was 3.2 for the energy condition and 1.1 for the non-energy condition.

In figure 6, the distribution of the answers is presented for each shape type in (a) energy (b) non-energy condition.
6. The interquartile range (IQR), the difference between the 75th and 25th percentile of the data, is chosen as a dispersion measure to examine the distribution of the responses since it is robust to outliers. In the case of the energy condition, the responses for pentagon and hexagon have an IQR of 1 and 4, respectively, while the other shapes have 0 IQR in their responses. For the non-energy condition, the IQRs for the shapes are 2, 2, 3, and 3 respectively. One can observe the more balanced distribution which can be considered as a sign of randomness in the responses for the non-energy condition in figure 6 and comparing IQR values. The non-balanced responses for circle, triangle, and rectangle in the energy condition can be interpreted as conscious choices rather than random responses.

Another interesting fact from figure 6 is the distribution of the incorrect responses to pentagon and hexagon in energy condition, figure 6(b). The ordering of the shapes with respect to decreasing probabilities among the incorrect responses to pentagon turns out to be hexagon, rectangle, triangle, circle while it is circle, pentagon, rectangle and triangle for hexagon. The consistency in the ordering can be interpreted as the ability to detect the approximate silhouette of the complex shapes even though the exact shape could not be detected with certainty. Circle having the highest number of responses to hexagon can be expected since the increasing number of edges for a shape converges to a circle and hence the higher the number of edges the more difficult it is to distinguish it from the circle.

7 Conclusions

Virtual coupling is one of the most widely used approaches for haptic rendering. The existing uses of virtual coupling during exploration of a surface, however, is not sufficient for providing comprehensive information about structures beneath a surface.

In order to address this problem, we have proposed casting a ray through an object in the proxy-probe direction used to update the virtual coupling spring stiffness. This allows detection of rigid bodies obscured by other rigid bodies beneath the surface by approaching them from different directions, in other words finding a direction along which they are not obscured. We have also incorporated an energy conservation component both for increased stability and to maintain a correct, physics-based interaction.

The user studies showed a high percentage of success for the task of locating an obscured rigid box by another rigid flat surface. We consider this as a promising result for further development in medical field for purposes such as detection of broken bones and cracks covered by fat layer or tumours deep beneath the soft tissue.

The energy conservation, providing gradient information, was tested in a task of shape perception through a surface. The results showed a significantly improved performance in distinguishing shapes like circle, triangle and rectangle for the energy conservation case. No significant improvement was observed for more complex shapes such as pentagon and hexagon. The consistent distribution of the incorrect responses in the case of complex shapes, however, can be interpreted as simply a sign that the test subjects are able to detect the approximate silhouette of the shapes.

Being able to feel underlying structures and surface properties simultaneously has potential in various contexts. Deformation simulation with force feedback, for instance, is a very challenging field and requires the use of computationally demanding techniques like Finite Element Modelling (FEM) with high resolution mesh structures in order to achieve realism. Our algorithm, AVC, being simpler to calculate allows use of much higher resolutions, although it is not able to compete with FEM in terms of realism. Considering the importance of resolution for realism, our algorithm might be considered either as an alternative or complement to these demanding techniques. In the case of exploring soft tissues for diagnosis of the presence of a tumour obscured by healthy tissue, feeling the presence or absence of the tumour may be more crucial than experiencing realistic forces. The advantage of our technique over the traditional deformation simulation algorithms is its simplicity and performance. In addition, the use of our technique is not limited to deformable objects but can be combined with any surface rendering algorithm including texture and friction rendering.

As future work, we intend to test the algorithms with medical data incorporating physical properties of the volume along the rays in the direction of proxy-probe and combine AVC with mesh-based deformation simulation algorithms for optimization purposes.

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